F(R) SUPERGRAVITY [1]

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Abstract

We review the F(R) supergravity recently proposed in Phys. Lett. B674(2009)59 and Class. and Quantum Grav. 26(2009)135006. Our construction supersymmetrizes popular f(R) theories of modified gravity in four spacetime dimensions. We use curved superspace of N=1 Poincaré supergravity in its minimal (2nd order) formulation so that our F(R) supergravity action is manifestly invariant under local N=1 supersymmetry. We prove that the F(R) supergravity is classically equivalent to the standard N=1 Poincaré supergravity coupled to a dynamical chiral superfield, via a Legendre-Weyl transform in superspace. A Kähler potential, a superpotential and a scalar potential of the chiral superfield are governed by a single holomorphic function. We find the conditions of vanishing cosmological constant without fine-tuning, which define a no-scale F(R) supergravity.

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1 Introduction

According to the contemporary 'new standard cosmology' based on recent astronomical data, the main ingredient of our *universe* is (*invisible*) dark energy that contributes almost 3/4 of everything. Though the dark energy may simply be a vacuum energy, or a cosmological constant, it is still possible (and even natural) that it is dynamical, i.e. it varies with time. For example, in *quintessence* models, the dark energy is attributed to a dynamical scalar field with a slowly declining potential. It is appealing to high energy physics and string theory that have many scalar fields (at least, in theory).

The apparently different class of gravitational theories, whose Lagrangian f(R) is a function of the Ricci scalar R, are also considered for describing dynamical dark energy. Those phenomenological f(R) gravity models can easily 'explain' the observed accelerated expansion of our universe after replacing the Einstein-Hilbert Lagrangian (proportional to R) by a suitable function f(R).

Those issues are in the focus of this meeting 'Invisible Universe', so that I feel there is no need to elaborate and give references on the basic things.

In our recent papers [1] we constructed for the first time the modified supergravity theory that can be interpreted as the N=1 locally supersymmetric generalization of the f(R) gravity. Our modified supergravity is parameterized by a holomorphic function.

A supersymmetric extension of the f(R) gravity theories is non-trivial because, despite of the apparent presence of the higher derivatives, there should be no ghosts, potential instabilities are to be avoided, and the auxiliary freedom [2] is to be preserved. It is proved [1] that our modified supergravity action is classically equivalent to the *standard* N=1 Poincaré supergravity coupled to a dynamical chiral superfield whose Kähler potential and superpotential are dictated by a single holomorphic function (see below).

In Sec. 2 we briefly review the f(R) gravity models and recall a well known proof of their (classical) equivalence to the quintessence models. In Sec. 3 we provide our superpace notation and setup. In Sec. 4 we prove the classical equivalence between our modified supergravity and the supersymmetric quintessence model of a single chiral superfield. In Sec. 5 we introduce a no-scale modified supergravity via its equivalent quintessence representation.

2 f(R) gravity and quintessence

An f(R) gravity [3] is specified by the action

$$S_f = -\frac{1}{2\kappa^2}\int d^4x\, f(R) \tag{2.1}$$

where R is the Ricci scalar curvature of a metric $g_{\mu\nu}(x)$, and κ is the gravitational coupling constant, $\kappa^2 = 8\pi G_N$. A matter action S_m minimally coupled to the

metric, can be added to eq. (2.1).

The gravitational equations of motion derived from the action (2.1) read

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + g_{\mu\nu}\Box f'(R) - \nabla_\mu\nabla_\nu f'(R) = 0 \tag{2.2}$$

where the primes denote differentiation. The equations of motion are thus the 4th-order differintial equations with respect to the metric (ie. with the higher derivatives). Taking the trace of eq. (2.2) yields

$$\Box f'(R) + \tfrac{1}{3}f'(R)R - \tfrac{2}{3}f(R) = 0 \tag{2.3}$$

Hence, in contrast to General Relativity with $f'(R) = const.$, in f(R) gravity the field $\omega = f'(R)$ is dynamical, thus representing an independent propagating (scalar) degree of freedom. In terms of the fields $(g_{\mu\nu}, \omega)$ the equations of motion are of the 2nd order.

The easiest way to make a connection between f(R) gravity and scalar-tensor gravity is to apply a Legendre-Weyl transform [4]. The action (2.1) is classically equivalent to

$$S_A = \frac{-1}{2\kappa^2}\int d^4x\,\sqrt{-g}\,\{AR - V(A)\} \tag{2.4}$$

where the real scalar $A(x)$ is related to the scalar curvature R by the Legendre transformation

$$R = V'(A) \qquad \text{and} \qquad f(R) = RA(R) - V(A(R)) \tag{2.5}$$

A Weyl transformation of the metric

$$g_{\mu\nu}(x) \to \exp\left[\frac{2\kappa\phi(x)}{\sqrt{6}}\right] g_{\mu\nu}(x) \tag{2.6}$$

with an arbitrary field parameter $\phi(x)$ yields

$$\sqrt{-g}\,R \to \sqrt{-g}\,\exp\left[\frac{2\kappa\phi(x)}{\sqrt{6}}\right]\left\{R - \sqrt{\frac{6}{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)\kappa - \kappa^2 g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right\} \tag{2.7}$$

Hence, when choosing

$$A(\kappa\phi) = \exp\left[\frac{-2\kappa\phi(x)}{\sqrt{6}}\right] \tag{2.8}$$

and ignoring the total derivative, we can rewrite the action (2.4) to the form

$$S_\phi = \int d^4x\,\sqrt{-g}\left\{\frac{-R}{2\kappa^2} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2\kappa^2}\exp\left[\frac{4\kappa\phi(x)}{\sqrt{6}}\right]V(A(\kappa\phi))\right\} \tag{2.9}$$

in terms of the physical (and canonically normalized) scalar field $\phi(x)$.

The applicability of the Legendre-Weyl transform implies the invertibility of the function $f'(R)$, ie. (locally) $f''(R) \neq 0$. Actually, one has to demand $f''(R) > 0$ in order to avoid a tachyon-like instability [5]. In addition, after the Weyl transform (2.6), the gravity-coupled matter fields in S_m become conformally coupled to ω. Hence, some stabilization mechanism is needed for ω.

Of course, in order to be phenomenologically viable, the f(R) gravity models are supposed to pass various tests coming from solar system and high-energy physics experiments — see eg., ref. [6].

3 Superspace supergravity

A concise and manifestly supersymmetric description of supergravity is given by superspace [7]. In this section we provide just a few equations, in order to set up our notation.

The chiral superspace density (in the supersymmetric gauge-fixed form) is

$$\mathcal{E}(x,\theta) = e(x)\left[1 - 2i\theta\sigma_a\bar{\psi}^a(x) + \theta^2 B(x)\right] \ , \tag{3.10}$$

where $e = \sqrt{-\det g_{\mu\nu}}$, $g_{\mu\nu}$ is a spacetime metric, $\psi^a_\alpha = e^a_\mu \psi^\mu_\alpha$ is a chiral gravitino, $B = S - iP$ is the complex scalar auxiliary field. We use the lower case middle greek letters $\mu, \nu, \ldots = 0,1,2,3$ for curved spacetime vector indices, the lower case early latin letters $a, b, \ldots = 0,1,2,3$ for flat (target) space vector indices, and the lower case early greek letters $\alpha, \beta, \ldots = 1,2$ for chiral spinor indices.

The solution of the superspace Bianchi identitiies and the constraints defining the N=1 Poincaré-type minimal supergravity results in only three relevant superfields $\mathcal{R}$, $\mathcal{G}_a$ and $\mathcal{W}_{\alpha\beta\gamma}$ (as parts of the supertorsion), subject to the off-shell relations [7]

$$\mathcal{G}_a = \bar{\mathcal{G}}_a \ , \qquad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)} \ , \qquad \bar{\nabla}_{\dot{\alpha}}\mathcal{R} = \bar{\nabla}_{\dot{\alpha}}\mathcal{W}_{\alpha\beta\gamma} = 0 \ , \tag{3.11}$$

and

$$\bar{\nabla}^{\dot{\alpha}}\mathcal{G}_{\alpha\dot{\alpha}} = \nabla_\alpha \mathcal{R} \ , \qquad \nabla^\gamma \mathcal{W}_{\alpha\beta\gamma} = \tfrac{i}{2}\nabla_\alpha{}^{\dot{\alpha}}\mathcal{G}_{\beta\dot{\alpha}} + \tfrac{i}{2}\nabla_\beta{}^{\dot{\alpha}}\mathcal{G}_{\alpha\dot{\alpha}} \ , \tag{3.12}$$

where $(\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}.\nabla_{\alpha\dot{\alpha}})$ represent the curved superspace N=1 supercovariant derivatives, and bars denote complex conjugation.

The covariantly chiral complex scalar superfield $\mathcal{R}$ has the scalar curvature R as the coefficient at its θ^2 term, the real vector superfield $\mathcal{G}_{\alpha\dot{\alpha}}$ has the traceless Ricci tensor, $R_{\mu\nu} + R_{\nu\mu} - \frac{1}{2}g_{\mu\nu}R$, as the coefficient at its $\theta\sigma^a\bar{\theta}$ term, whereas the covariantly chiral, complex, totally symmetric, fermionic superfield $\mathcal{W}_{\alpha\beta\gamma}$ has the Weyl tensor $W_{\alpha\beta\gamma\delta}$ as the coefficient at its linear θ^δ-dependent term.

A generic supergravity Lagrangian (eg., representing the supergravitational part of the superstring effective action) is

$$\mathcal{L} = \mathcal{L}(\mathcal{R}, \mathcal{G}, \mathcal{W}, \ldots) \tag{3.13}$$

where the dots stand for arbitrary covariant derivatives of the supergravity superfields. We would like to concentrate on the particular sector of the theory (3.13), by ignoring the tensor superfields $\mathcal{W}_{\alpha\beta\gamma}$ and $\mathcal{G}_{\alpha\dot{\alpha}}$, and the derivatives of the scalar superfield $\mathcal{R}$.

The F(R) supergravity action proposed in ref. [1] reads [3]

$$S_F = \int d^4x d^2\theta \, \mathcal{E} F(\mathcal{R}) + \text{H.c.} \tag{3.14}$$

with some holomorphic function $F(\mathcal{R})$. Besides manifest local N=1 supersymmetry, the action (3.14) also possess the auxiliary freedom [2], since the auxiliary field B does not propagate. It distinguishes the action (3.14) from other possible truncations of eq. (3.13). In addition, the action (3.14) gives rise to a spacetime torsion.

4 Super-Legendre-Weyl-Kähler transform

The superfield action (3.14) is classically equivalent to another action

$$S_V = \int d^4x d^2\theta \, \mathcal{E} \left[\mathcal{Z}\mathcal{R} - V(\mathcal{Z}) \right] + \text{H.c.} \tag{4.15}$$

where we have introduced the covariantly chiral superfield $\mathcal{Z}$ as a Lagrange multiplier. Varying the action (4.15) with respect to $\mathcal{Z}$ gives back the original action (3.14) provided that

$$F(\mathcal{R}) = \mathcal{R}\mathcal{Z}(\mathcal{R}) - V(\mathcal{Z}(\mathcal{R})) \tag{4.16}$$

where the function $\mathcal{Z}(\mathcal{R})$ is defined by inverting the function

$$\mathcal{R} = V'(\mathcal{Z}) \tag{4.17}$$

Equations (4.16) and (4.17) define the superfield Legendre transform. They imply further relations

$$F'(\mathcal{R}) = Z(\mathcal{R}) \qquad \text{and} \qquad F''(\mathcal{R}) = Z'(\mathcal{R}) = \frac{1}{V''(\mathcal{Z}(\mathcal{R}))} \tag{4.18}$$

where $V'' = d^2V/d\mathcal{Z}^2$. The second formula (4.18) is the duality relation between the supergravitational function F and the chiral superpotential V.

A super-Weyl transform of the superfeld acton (4.15) can be done entirely in superspace, ie. with manifest local N=1 supersymmetry. In terms of field components, the super-Weyl transform amounts to a Weyl transform, a chiral rotation and a (superconformal) S-supersymmetry transformation [8].

[3]We hide the gravitational coupling constant for simplicity.

The chiral density superfield $\mathcal{E}$ is a chiral compensator of the super-Weyl transformations

$$\mathcal{E} \to e^{3\Phi}\mathcal{E} \tag{4.19}$$

whose parameter Φ is an arbitrary covariantly chiral superfield, $\bar{\nabla}_{\dot{\alpha}}\Phi = 0$. Under the transformation (4.19) the covariantly chiral superfield $\mathcal{R}$ transforms as

$$\mathcal{R} \to e^{-2\Phi}\left(\mathcal{R} - \tfrac{1}{4}\bar{\nabla}^2\right)e^{\bar{\Phi}} \tag{4.20}$$

The super-Weyl chiral superfield parameter Φ can be traded for the chiral Lagrange multiplier $\mathcal{Z}$ by using a generic gauge condition [4]

$$\mathcal{Z} = \mathcal{Z}(\Phi) \tag{4.21}$$

where $\mathcal{Z}(\Phi)$ is an arbitrary (holomorphic) function of Φ. Then the super-Weyl transform of the acton (4.15) results in the classically equivalent action

$$S_\Phi = \int d^4x d^4\theta\, E^{-1}e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})\right] + \left[-\int d^4x d^2\theta\, \mathcal{E}e^{3\Phi}V(\mathcal{Z}(\Phi)) + \text{H.c.}\right] \tag{4.22}$$

where we have introduced the full supergravity supervielbein E^{-1} [7].

Equation (4.22) has the standard form of the action of a chiral matter superfield coupled to supergravity [7],

$$S[\Phi,\bar{\Phi}] = \int d^4x d^4\theta\, E^{-1}\Omega(\Phi,\bar{\Phi}) + \left[\int d^4x d^2\theta\, \mathcal{E}P(\Phi) + \text{H.c.}\right] \tag{4.23}$$

in terms of a 'Kähler' potential $\Omega(\Phi,\bar{\Phi})$ and a chiral superpotential $P(\Phi)$. In our case (4.22) we find

$$\begin{aligned} \Omega(\Phi,\bar{\Phi}) &= e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi})\right] \ , \\ P(\Phi) &= -\, e^{3\Phi}V(\mathcal{Z}(\Phi)) \end{aligned} \tag{4.24}$$

The truly Kähler potential $K(\Phi,\bar{\Phi})$ is given by [7]

$$K = -3\ln(-\frac{\Omega}{3}) \quad \text{or} \quad \Omega = -3e^{-K/3} \ , \tag{4.25}$$

because of the invariance of the action (4.23) under the supersymmetric Kähler-Weyl transformations

$$K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}) \ , \quad \mathcal{E} \to e^{\Lambda(\Phi)}\mathcal{E} \ , \quad P(\Phi) \to -e^{-\Lambda(\Phi)}P(\Phi) \tag{4.26}$$

with an arbitrary chiral superfield parameter $\Lambda(\Phi)$.

[4] In the first paper of ref. [1] we used the particular gauge $\xi\Phi = \ln\mathcal{Z}$ with some number ξ.

The scalar potential (in components) is given by the standard formula [9]

$$\mathcal{V}(\phi,\bar{\phi}) = e^{\Omega}\left\{ \left| \frac{\partial P}{\partial \Phi} + \frac{\partial \Omega}{\partial \Phi} P \right|^2 - 3\left|P\right|^2 \right\} \Bigg| \tag{4.27}$$

where all superfields are restricted to their leading field components, $\Phi| = \phi(x)$. Equation (4.27) can be simplified by making use of the Kähler-Weyl invariance (4.26) that allows us to choose the gauge

$$P = 1 \tag{4.28}$$

It is equivalent to the well known fact that the scalar potential (4.27) is actually governed by the single (Kähler-Weyl invariant) potential [7]

$$G(\Phi,\bar{\Phi}) = \Omega + \ln P + \ln \bar{P} \tag{4.29}$$

In our case (4.24) we have

$$G = e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi}) \right] + 3(\Phi+\bar{\Phi}) + \ln(-V(\mathcal{Z}(\Phi)) + \ln(-\bar{V}(\bar{\mathcal{Z}}(\bar{\Phi})) \tag{4.30}$$

Let's now specify our gauge (4.21) by choosing the condition

$$3\Phi + \ln(-V(\mathcal{Z}(\Phi)) = 0 \quad \text{or} \quad V(\mathcal{Z}(\Phi)) = -e^{-3\Phi} \tag{4.31}$$

that is equivalent to eq. (4.28). Then the potential (4.30) gets simplified to

$$G = \Omega = e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi) + \bar{\mathcal{Z}}(\bar{\Phi}) \right] \tag{4.32}$$

Equations (4.16), (4.17) and (4.32) are the simple one-to-one algebraic relations between a holomorphic function $F(\mathcal{R})$ in our modified supergravity action (3.14) and a holomorphic function $\mathcal{Z}(\Phi)$ entering the potential (4.32) and defining the scalar potential (4.27) as

$$\mathcal{V} = e^{G}\left[\left(\frac{\partial^2 G}{\partial\Phi\partial\bar{\Phi}} \right)^{-1} \frac{\partial G}{\partial \Phi}\frac{\partial G}{\partial \bar{\Phi}} - 3 \right] \Bigg| \tag{4.33}$$

in the classically equivalent scalar-tensor supergravity. The latter can used for embedding the standard slow-roll inflation into supergravity.

In the next Sec. 5 we discuss eqs. (4.32) and (4.33) in terms of a function $\mathcal{Z}(\Phi)$.

5 No-scale supergravity

A no-scale supergravity arises by demanding the scalar potential (4.33) to vanish. It results in the vanishing cosmological constant without fine-tuning [10]. The

no-scale supergravity potential G has to obey the non-linear 2nd-order partial differential equation

$$3\frac{\partial^2 G}{\partial\Phi\partial\bar{\Phi}} = \frac{\partial G}{\partial\Phi}\frac{\partial G}{\partial\bar{\Phi}} \tag{5.34}$$

A gravitino mass $m_{3/2}$ is given by the vacuum expectation value [7]

$$m_{3/2} = \left\langle e^{G/2} \right\rangle \tag{5.35}$$

so that the spontaneous supersymmetry breaking scale can be chosen at will.

The well known exact solution to eq. (5.34) is given by

$$G = -3\ln(\Phi + \bar{\Phi}) \tag{5.36}$$

In the recent literature, the no-scale solution (5.36) is usually modified by other terms, in order to describe a universe with positive cosmological constant.

Just to appreciate the difference between the standard no-scale supergravity solution and our case, it is worth noticing that the ansatz (5.36) is inconsistent with our potential (4.32) by any choice of the function $\mathcal{Z}$. Demanding eq. (5.34) in our case gives rise to the 1st-order non-linear partial differential equation

$$3\left(e^{\bar{\Phi}}X' + e^{\Phi}\bar{X}'\right) = \left|e^{\bar{\Phi}}X' + e^{\Phi}\bar{X}\right|^2 \tag{5.37}$$

where we have introduced the notation

$$\mathcal{Z}(\Phi) = e^{-\Phi}X(\Phi)\ , \qquad X' = \frac{dX}{d\Phi} \tag{5.38}$$

The gravitino mass (5.35) is given by

$$m_{3/2} = \left\langle \exp\frac{1}{2}\left(e^{\bar{\Phi}}X + e^{\Phi}\bar{X}\right)\right\rangle \tag{5.39}$$

We are not aware of any analytic holomorphic exact solution to eq. (5.37). Should it obey a holomorphic differential equation of the form

$$X' = e^{\Phi}g(X,\Phi) \tag{5.40}$$

with a holomorphic function $g(X,\Phi)$, eq. (5.37) gives rise to the functional equation

$$3\left(g + \bar{g}\right) = \left|e^{\bar{\Phi}}g + \bar{X}\right|^2 \tag{5.41}$$

When being restricted to the real variables $\Phi = \bar{\Phi} \equiv y$ and $X = \bar{X} \equiv x$, eq. (5.37) reads

$$6x' = e^{y}(x' + x)^2\ , \quad \text{where} \quad x' = \frac{dx}{dy} \tag{5.42}$$

This equation can be integrated after the change of variables [5]

$$x = e^{-y}u \tag{5.43}$$

which leads to the quadratic equation with respect to $u' = du/dy$

$$(u')^2 - 6u' + 6u = 0 \tag{5.44}$$

Its solution reads

$$y = \int^u \frac{d\xi}{3 \pm \sqrt{3(3-2\xi)}} = \mp \sqrt{1 - \tfrac{2}{3}u} + \ln\left(\sqrt{3(3-2u)} \pm 3\right) + C. \tag{5.45}$$

6 Instead of Conclusion

Since the f(R) gravities are merely phenomenological models, it would be great to generate them from a fundamental theory, like strings or M-theory. The latter need supersymmetry for their consistency, so that our construction of F(R) supergravities is just the first step in that direction. The leading complex scalar field component of the chiral superfield Φ may be identified with a dilaton-axion field [1, 11]. As is clear from a generic form (3.13) of the gravitational effective action from any fundamental theory, both f(R) gravities and F(R) supergravities are not universal because of the presence of the other terms depending upon the Weyl and Ricci curvature. Hence, the f(R) gravities and supergravities should be considered as merely the toy models for limited purposes. As regards other modified supergravities, see eg., refs. [12, 13].

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